CIRCUIT THEORY

CIRCUIT TRANSIENTS

SYED HASAN SAEED

REFERENCE BOOKS

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- Networks And Systems, Ashfaq Husain, Khanna Book Publishing Co (P) Ltd. Delhi.
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- Network Analysis, M.E. Van Valkenburg, PHI Learning Private limited, New Delhi.
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CIRCUIT TRANSIENTS

When a circuit is switched from one condition to another condition, there is a transitional period. This transitional period is known as *transient period*. During the transient period there is a change in applied voltage (by removing or introducing the source) or in one of the circuit elements. The behavior of the voltage or current when it is changed from one condition to another is called ***transient state***. The time taken by the circuit to change from one steady state to another is called ***transient time***.

After the transient period (known as steady state response) the response becomes stable and independent of time.

During the transient state the circuit should satisfy the Kirchhoff's law.

The circuit may have energy storage elements (inductor and capacitor). The equations can be obtained by applying Kirchhoff's law. Theses equations are linear integral differential equations with constant coefficient.

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The solution of these equations consists of two parts

(i) The solution of equations with energy sources set equal to zero. This is known as complementary or transient function. The response goes to zero relatively in a short time and hence it is called transient part of the solution.

(ii) This part is known as particular integral or steady state solution of the response.

Therefore, ***Total Response = Transient Response + Steady State Response***

For complete solution of the circuit, initial conditions are used for determining the arbitrary constants of the differential equations.

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INITIAL CONDITIONS:

Initial conditions are used for determining the arbitrary constants for the solution of differential equations.

Initial conditions gives the information about the behavior of the circuit elements at the time of switching.

- The switching time is generally taken as $t = 0$, known as reference time.
- The time just before switching is given by $t = 0_-$
- The time just after the switching is given by $t = 0_+$
- The current and voltage just before the switching are given by $i(0_-)$ and $v(0_-)$ respectively. These values must known.
- After switching new currents and voltages may appear in the network at $t = 0_+$
- The current and voltage just after the switching are given by $i(0_+)$ and $v(0_+)$ respectively.

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INITIAL CONDITIONS IN CIRCUIT ELEMENTS:

(i) Resistor: consider the following circuit

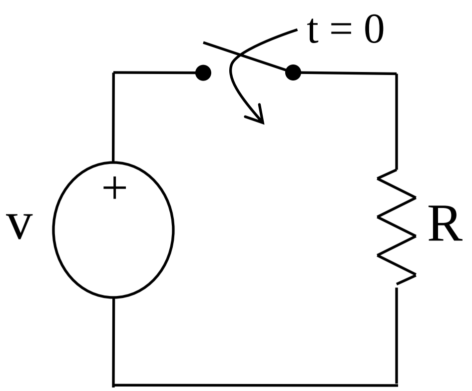

Fig. 1a

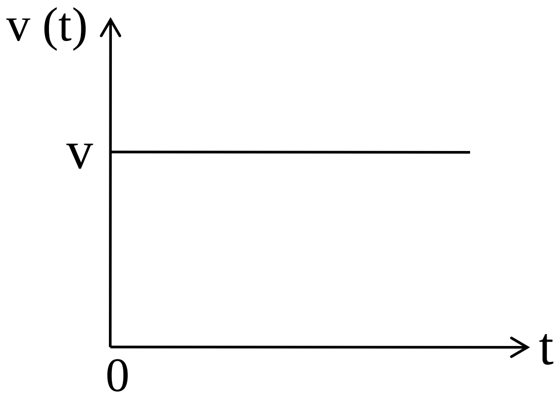

Fig. 1b

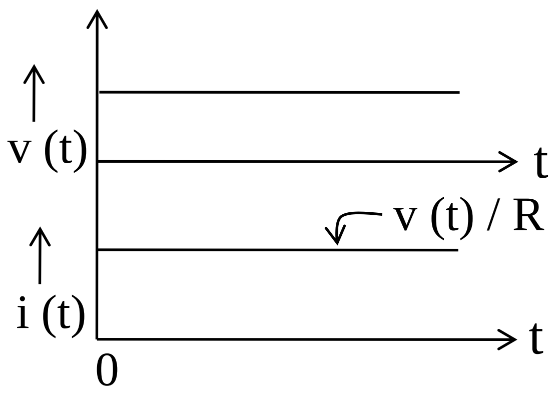

Fig. 1c

- If by closing the switch at t = 0, apply the step voltage across the resistor , then the current have the same waveform as input and the final value is given by v/R at t = 0.
- Thus, the resistor accepts any change across it instantaneously that is the current through the resistor will change instantaneously if the voltage changes instantaneously and vice versa. There is no transient period in case of resistor.

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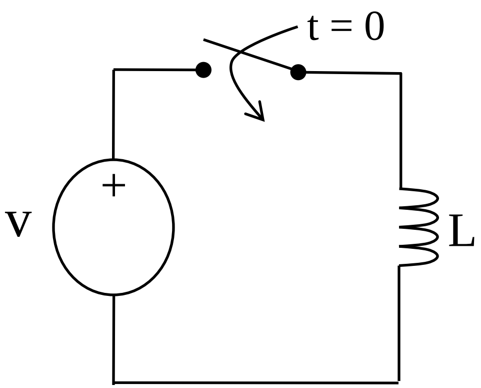
(ii) Inductor: Inductance is the property of material which opposes any changes in current through it, so current through inductor cannot change instantaneously. So when an energy source is connected then the current through the inductor before and after the change of position of swtich remains the same.

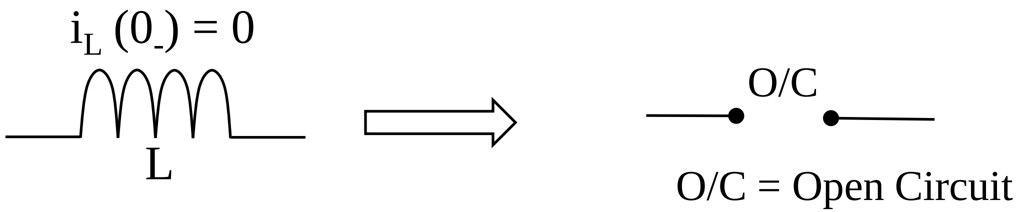
i.e $$\mathbf{i_L(0_-) = i_L(0_+)}$$

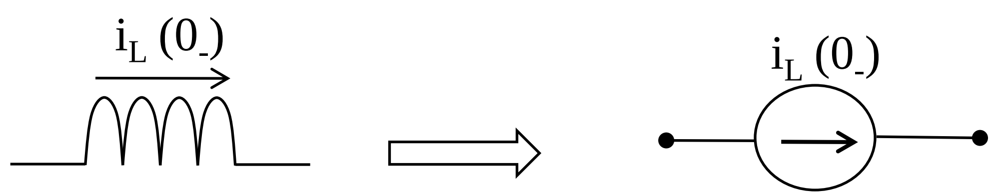
Therefore, when energy source is suddenly connected to an inductor will not cause current to flow ***initially and inductor will act as an open circuit.***

If the inductor was carrying some current i_L (0-) at t = 0- then at t = 0_+ it acts like a current source of value $i_L(0_-)$



$$V = L\frac{di}{dt}$$





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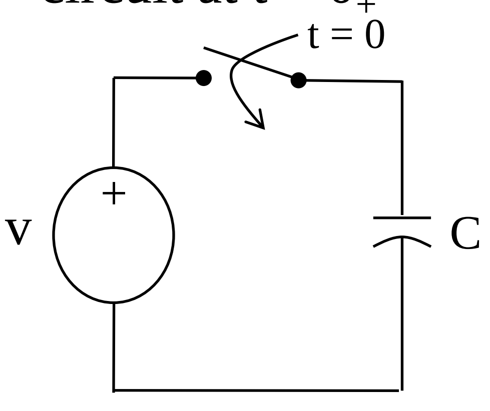
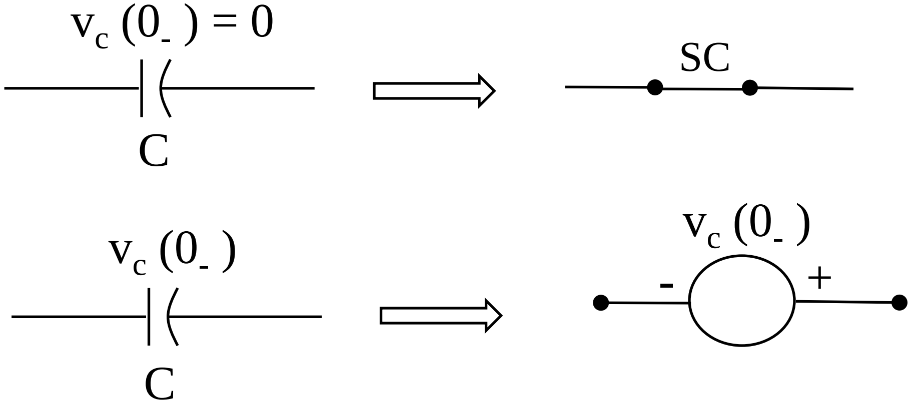
(iii) Capacitor: Capacitance is the property of material which opposes any changes in voltage across it, so voltage across capacitance cannot change instantaneously. If an uncharged capacitor is switched on to a DC source the current will flow instantaneously and the capacitor will act as short circuit (V = 0), then at t = 0

$$\mathbf{v_c(0_-) = v_c(0_+)} \qquad \textbf{or} \qquad \mathbf{q_c(0_-) = q_c(0_+)}$$

Where q_c is the charge on capacitor.

In other words, if energy source is suddenly applied at t = 0 to any uncharged capacitor by closing the switch, then no voltage will appear across the capacitor at $t = 0_+$ because $v_c(0_-)$ was zero. So capacitor will act like short circuit at $t = 0_+$





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STEADY STATE CONDITIONS:

Steady state conditions can be obtained for all three elements in network in which the final value for voltage and current is a constant.

The final steady state equivalent networks are derived from basic relationships. Under steady state condition, the derivatives have zero values. Hence, an inductor acts short-circuited and a capacitor acts as open-circuited for constant steady state value.

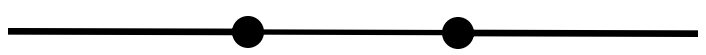
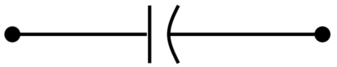
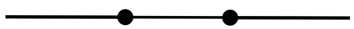
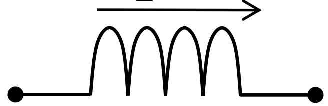
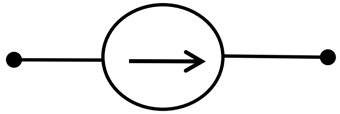
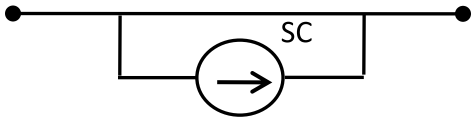
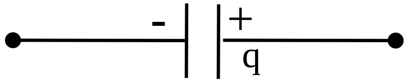
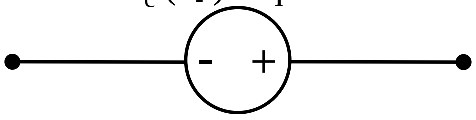
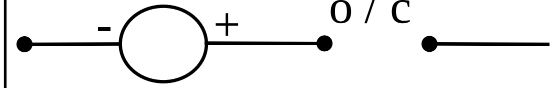
Procedure To Find The Final Conditions:

Step 1: Draw the equivalent circuit at $t = 0_+$, by replacing the inductors with open circuit or with constant current source having the current flowing at

$t = 0_-$ and capacitor with short circuit or with constant voltage source if there is any initial charged voltage at $t = 0_-$. Resistors are left in the network without any change. Evaluate the initial values.

Step 2: To find $\frac{di}{dt}$ and $\frac{dv}{dt}$ at $t = 0_+$, write either KCL or KVL expressions for all values of t and substitute theses values of 'i' or 'v' at $t = 0_+$.

Elements And Their Equivalent Circuits:

Elements	Equivalent Circuit at t = 0_+	Equivalent Circuit at t = ∞
R 	R 	R 
L 	o / c 	s / c 
C 	s / c 	o / c 
$i_L\ (0_-)$ 	$i_L\ (0_-)$ 	 sc, $i_L\ (0_-)$
$v_c\ (0_-)$  -, +, q	$v_c\ (0_-) = q/C$  -, +	 -, +, o / c, $v_c\ (0_-)$

Reference: Electrical Circuit Analysis, S. Sivanagaraju, g. Kishor, C. Srinivasa Rao, CENGAGE Learning
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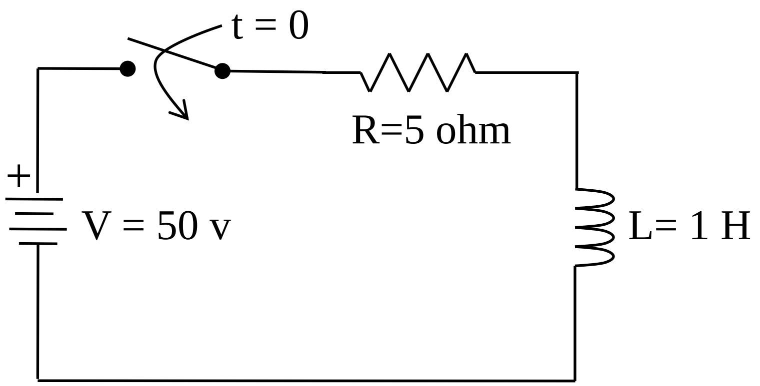
EXAMPLE 1: Find i and di/dt at $t = 0_+$ if switch is closed at t = 0.



Current through inductor before closing the switch $i(0_-) = 0$

$\therefore \quad i(0_+) = 0 \qquad\qquad \because i(0_-) = i(0_+) = 0$

Apply KVL in the given circuit

$$V = R\,i + L\,\frac{di}{dt}$$

$$50 = 5\,i(0_+) + 1 * \frac{di(0_+)}{dt}$$

$$\therefore \frac{di(0_+)}{dt} = 50\ A/s$$

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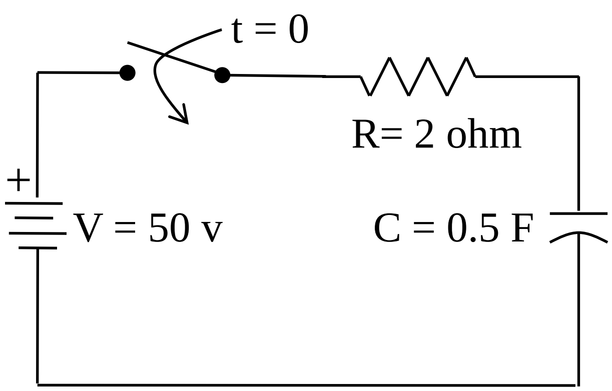
EXAMPLE 2:



If switch is closed at $t = 0$

Find $i\,(0_+)$ and $\dfrac{di\,(0_+)}{dt}$

SOLUTION :

At $t = 0_-$ the capacitor voltage $v_c\,(0_-) = 0$

$\therefore v_c\,(0_+) = 0$ because capacitor was uncharged

$$V = R\,i + \frac{1}{C}\int i\,dt \quad \_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

$$V = R\,i\,(0_+) + \frac{1}{C}\int i\,(0_+)\,dt \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (2)$$

$$v_c\,(0_+) = \frac{1}{C}\int i\,(0_+)\,dt = 0$$

$$\therefore 50 = 2\,i\,(0_+) + 0$$

$$i\,(0_+) = 25\text{ A}$$

$$\frac{dv\,(0_+)}{dt} = R\,\frac{di\,(0_+)}{dt} + \frac{1}{C}\,i\,(0_+) \qquad \text{(differentiate equation (2))}$$

$$0 = 2\,\frac{di\,(0_+)}{dt} + \frac{1}{2} * 25$$

$$\frac{di\,(0_+)}{dt} = -\,25\text{ Amp / sec}$$

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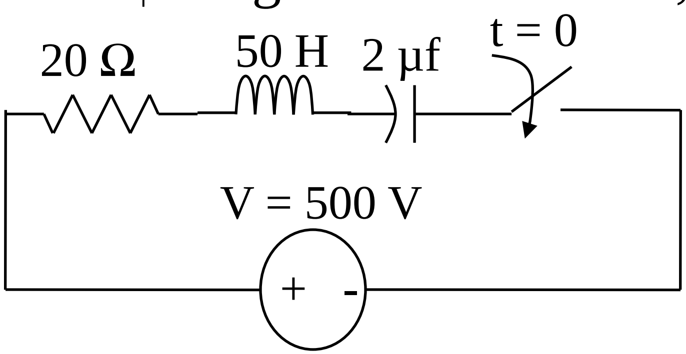
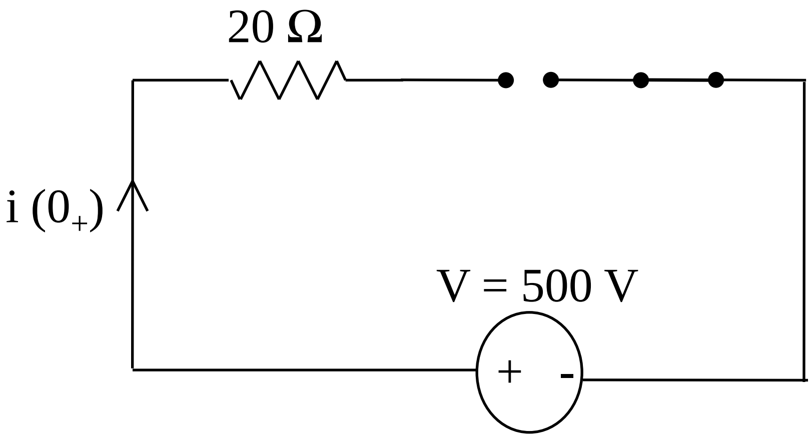
EXAMPLE 3: Calculate the following at $t = 0_+$ for given RLC circuit, if the switch is closed at $t = 0$

$$i(0_+), \frac{di(0_+)}{dt}, \frac{d^2 i(0_+)}{dt^2}$$



SOLUTION:

Equivalent Circuit at $t = 0_+$



Before closing the switch $i(0_-) = 0$

$\therefore i(0_-) = 0 = i(0_+)$

Inductor will act like open circuit at $i(0_+)$

Apply KVL

$$V = R\,i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{\_\_\_\_\_\_\_\_(1)}$$

At $i(0_-) = 0,\ V_C(0_-) = 0$

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Capacitor will act as short circuit at $t = 0_+$

$$\therefore \frac{1}{C}\int i\,dt = V_C = 0, \text{put in equation (1)}$$

so, equation (1) becomes

$$R\,i(0_+) + L\frac{di(0_+)}{dt} = V$$

$$20 * 0 + 50\frac{di(0_+)}{dt} = 500$$

$$\therefore \frac{di(0_+)}{dt} = \frac{500}{50} = 10\ A/sec$$

Differentiate equation (1)

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$$

at $t = 0_+$

$$R\frac{di(0_+)}{dt} + L\frac{d^2 i(0_+)}{dt^2} + \frac{1}{C}i(0_+) = 0$$

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Put the values of R, L and C, we get

$$20\frac{di(0_+)}{dt}+50\frac{d^2i(0_+)}{dt^2}+\frac{1}{2*10^{-6}}i(0_+)=0$$

Put the values of $i(0_+)$ and $\frac{di(0_+)}{dt}$, we get

$$20*10+50\frac{d^2i(0_+)}{dt^2}+0=0$$

$$\therefore\frac{d^2i(0_+)}{dt^2}=-4\ A/sec^2$$

Reference: Electrical Circuit Analysis, S. Sivanagaraju, g. Kishor, C. Srinivasa Rao, CENGAGE Learning
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Example 4: In the given circuit the switch is closed at t = 0. find the following values of

$$i_1, i_2, \frac{di_1}{dt}, \frac{di_2}{dt}, \frac{d^2i_1}{dt^2}, \frac{d^2i_2}{dt^2} \text{ at } t = 0_+$$

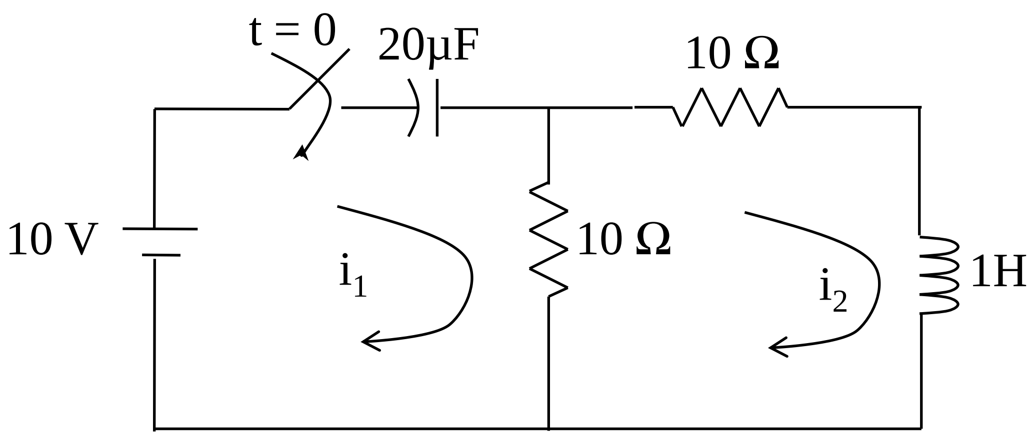

Fig. 1

Since switch was open at $t = 0_-$ so $i_1(0_-) = i_2(0_-) = 0$

When the switch is closed at $t = 0_+$, the current through inductor cannot change and also voltage across capacitor cannot change. Hence they act as open circuit and short circuit, respectively. Equivalent circuit shown in fig 2

$$i_2(0_+) = 0 \text{ and } i_1(0_+) = \frac{10}{10} = 1\,A$$

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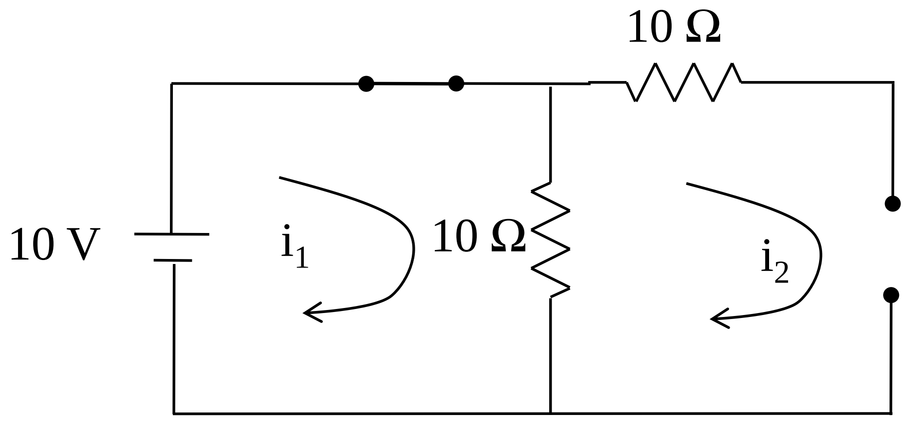
EQUIVALENT CIRCUIT:



Apply KVL in both meshes of fig.1

$$-\frac{1}{C}\int i_1 dt + 10(i_1 - i_2) = 10 \_\_\_\_\_ (1)$$

$$-10(i_2 - i_1) - 10 i_2 + \frac{di_2}{dt} = 0 \_\_\_\_\_ (2)$$

$$\text{or, } \frac{di_2}{dt} = 10 i_1 - 20 i_2$$

$$\frac{di_2(0_+)}{dt} = 10 i_1(0_+) - 20 i_2(0_+)$$

$$\frac{di_2(0_+)}{dt} = 10*1 - 20*0 = 10 \text{ A/sec}$$

Differentiate equation (1), we get

$$\frac{i_1(0_+)}{C} + 10\frac{di_1(0_+)}{dt} - 10\frac{di_2(0_+)}{dt} = 0$$

$$\frac{i_1(0_+)}{2*10^{-6}} + 10\frac{di_1(0_+)}{dt} - 10\frac{di_2(0_+)}{dt} = 0 \quad \_\_\_(3)$$

$$10*\frac{di_1(0_+)}{dt} = 10*\frac{di_2(0_+)}{dt} - \frac{1}{2*10^{-6}} \; *i_1$$

$$\frac{di_1(0_+)}{dt} = \frac{di_2(0_+)}{dt} - 0.5*10^5 \; *i_1$$

$$\frac{di_1(0_+)}{dt} = \frac{di_2(0_+)}{dt} - 0.5*10^5 \; *i_1 \quad \_\_\_\_(4)$$

put the values of $i_1(0_+)$, $\frac{di_2(0_+)}{dt}$

we get,

$$\frac{di_1(0_+)}{dt} = 10 - 0.5*10^5$$

$$\frac{di_1(0_+)}{dt} = -49990 \text{ A/sec}$$

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Differentiate equation (2), we get

$$\frac{d^2 i_2}{dt^2} = 10\frac{di_1}{dt} - 20\frac{di_2}{dt}$$

$$\frac{d^2 i_2}{dt^2} = 10*(-49990) - 20*10 = -500100 \text{ A/sec}^2$$

Differentiate equation (4) , we get

$$\frac{d^2 i_1}{dt^2} = \frac{d^2 i_2}{dt^2} - 0.5*10^5\frac{di_1}{dt}$$

$$\frac{d^2 i_1}{dt^2} = -500100 + 0.5*10^5 * 49990 = 2498999900 \text{ A/sec}^2$$

$$\frac{d^2 i_1}{dt^2} = 2.4989999*10^9 \text{ A/sec}^2$$

Reference: Electrical Circuit Analysis, S. Sivanagaraju, g. Kishor, C. Srinivasa Rao, CENGAGE Learning
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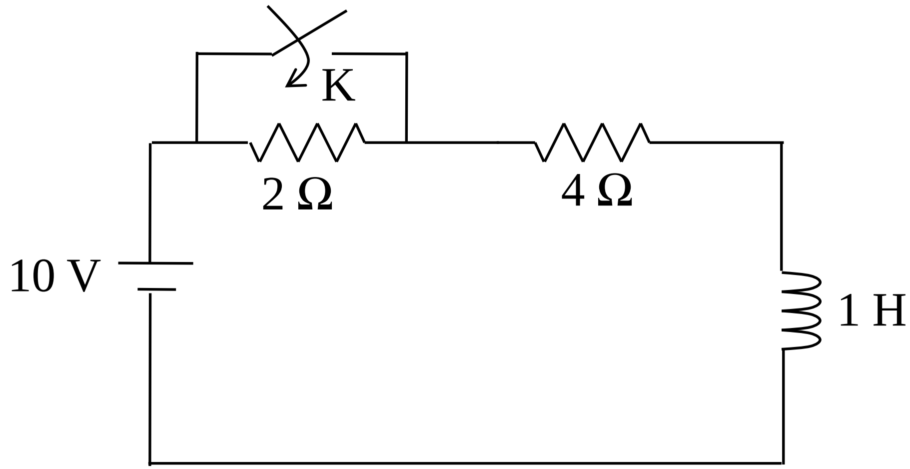
EXAMPLE 5: Find i (0_+) if switch is closed at t = 0 and steady state is reached with switch open. Draw circuit at $t \to 0_+$



Since steady state is reached, inductor will act as short circuit.

$$\therefore i(0_-) = \frac{10}{2+4} = \frac{10}{6} = 1.67 \text{ A}$$

Now, $i(0_+) = i(0_-) = 1.67$ A

Inductor will act as current source of 1.67 A. Circuit at $t = 0_+$ is shown in fig. 2

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Equivalent circuit at $t = 0_+$

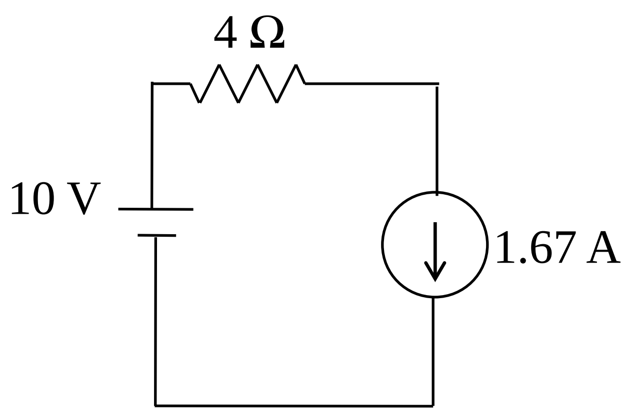

Fig. 2

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EXAMPLE 6: In the network shown in fig. 1, K is changed from position a to b at t = 0. Solve for

$$i, \frac{di}{dt}, \frac{d^2i}{dt^2}$$

at t = 0+, if R = 1000 ohm, L = 1 H, C = 0.1 μF and V = 100 V. Assume that the capacitor is uncharged.

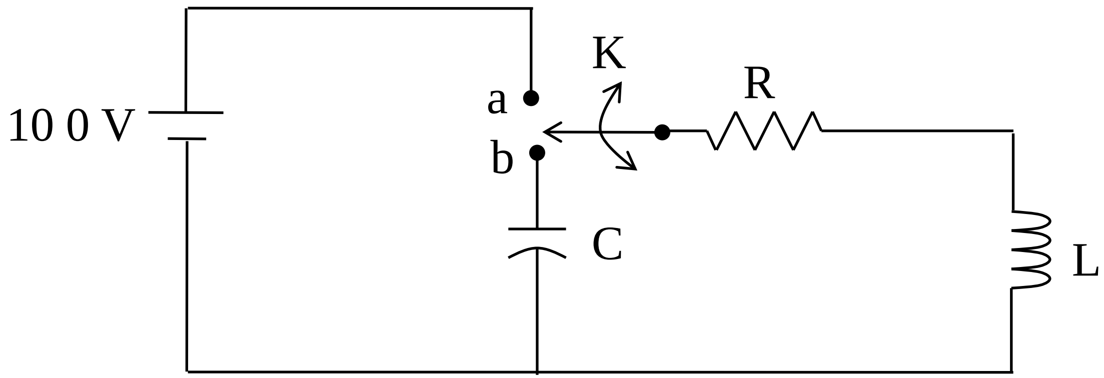

Fig. 1

SOLUTION: When switch is at position ‘a’, the inductor acts like short circuit.

$$i(0_-) = \frac{V}{R} = \frac{100}{1000} = 0.1\,A$$

$$\therefore i(0_+) = i(0_-) = 0.1\,A$$

i.e the inductor has a initial current of 0.1 A

When switch is moved from a to b, $i(0_+) = 0.1$

When K is at 'b', apply KVL

$$R\,i + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$R\,i(0_+) + L\frac{di(0_+)}{dt} + \frac{1}{C}\int i(0_+)\,dt = 0$$

Since capacitor was uncharged, then

$$\frac{1}{C}\int i(0_+)\,dt = v_C = 0$$

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$$\therefore 1000 * 0.1 + 1\frac{di}{dt}(0_+) + 0 = 0$$

$$\therefore \frac{di}{dt}(0_+) = -100 \text{ A/sec}$$

Differentiate equation (1)

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = 0$$

$$R\frac{di(0_+)}{dt} + L\frac{d^2 i(0_+)}{dt^2} + \frac{i(0_+)}{C} = 0$$

$$1000 * (-100) + 1\frac{d^2 i(0_+)}{dt^2} + \frac{0.1}{0.1 * 10^{-6}} = 0$$

$$\frac{d^2 i(0_+)}{dt^2} = -9 * 10^5 \text{ A/sec}^2$$

Network Analysis, M.E. Van Valkenburg, PHI Learning Private limited, New Delhi.

THANK YOU

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